7.1
Area of a Region Between Two Curves
Area of region between \( f \) and \( g \) = Area of region under \( f(x) \) - Area of region under \( g(x) \)

\[
\int_{a}^{b} [f(x) - g(x)] \, dx = \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx
\]
Ex. Find the area of the region bounded by the graphs of \( f(x) = x^2 + 2 \), \( g(x) = -x \), \( x = 0 \), and \( x = 1 \).

Area = Top curve – bottom curve

\[
A = \int_{0}^{1} [(x^2 + 2) - (-x)] \, dx = \int_{0}^{1} (x^2 + 2 + x) \, dx = \left[ \frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{0}^{1} = \frac{1}{3} + \frac{1}{2} + 2 = \frac{17}{6}
\]
Find the area of the region bounded by the graphs of \( f(x) = 2 - x^2 \) and \( g(x) = x \)

First, set \( f(x) = g(x) \) to find their points of intersection.

\[
2 - x^2 = x
\]

\[
0 = x^2 + x - 2
\]

\[
0 = (x + 2)(x - 1)
\]

\[
x = -2 \text{ and } x = 1
\]

\[
\int_{-2}^{1} [(2 - x^2) - x] \, dx = \text{fnInt}(2 - x^2 - x, x, -2, 1) = \frac{9}{2}
\]
Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$

Again, set $f(x) = g(x)$ to find their points of intersection.

$3x^3 - x^2 - 10x = -x^2 + 2x$

$3x^3 - 12x = 0$

$3x(x^2 - 4) = 0$

$x = 0, -2, 2$

Note that the two graphs switch at the origin.
Now, set up the two integrals and solve.

\[
\int_{-2}^{0} [f(x) - g(x)] \, dx + \int_{0}^{2} [g(x) - f(x)] \, dx
\]

\[
\int_{-2}^{0} (3x^3 - 12x) \, dx + \int_{0}^{2} (-3x^3 + 12x) \, dx = 24
\]
$$A = \int_{x_1}^{x_2} [(top \ curve) - (bottom \ curve)] \, dx$$

$$A = \int_{y_1}^{y_2} [(right \ curve) - (left \ curve)] \, dy$$
Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $y = x - 1$

\[ A = (3 - y^2) - (y + 1) \quad \text{from } -2 \text{ to } 1 \]

Area = Right - Left

\[ A = \int_{-2}^{1} [(3 - y^2) - (y + 1)] \, dy \]

\[ = \frac{9}{2} \]